The State of State MATH Standards

2005

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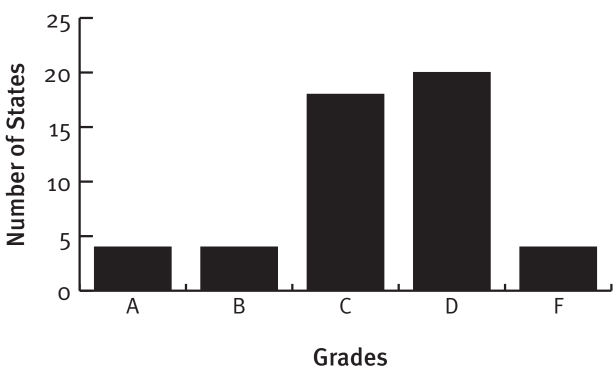
Criteria for Evaluation

State standards were judged on a 0-4 point scale on four criteria: clarity, content, reason, and negative qualities. In each case, 4 indicates excellent performance, 3 indicates good performance, 2 indicates mediocre performance, 1 indicates poor performance, and 0 indicates failing performance. More information about how grades were assigned is available in the "Methods and Procedures" section beginning on page 121.[7]

Clarity

Fig. 8: 2005 Grades for Clarity



State average: 1.85
Range: 0.33-3.83

States to watch:
California (3.83)
Indiana, Massachusetts (3.67)
Georgia (3.33)
Alabama, New Mexico (3.00)

States to shun:
Washington, Connecticut (0.33)
Missouri (0.67)
Delaware (0.83)

Clarity refers to the success the document has in achieving its own purpose, i.e., making clear to teachers, test developers, textbooks authors, and parents what the state desires. Clarity refers to more than the prose, however. The clarity grade is the average of three separate sub-categories:

1. *Clarity* of the language: The words and sentences themselves must be understandable, syntactically unambiguous, and without needless jargon.

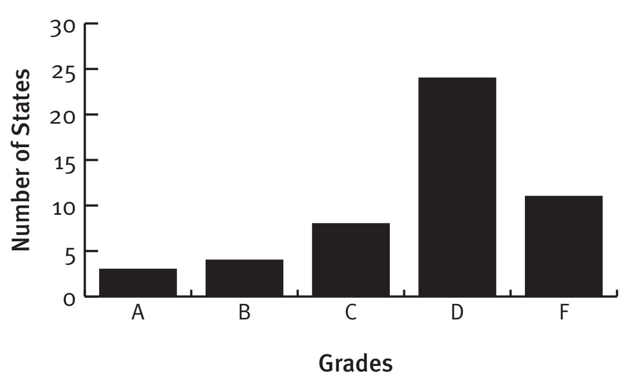
2. *Definiteness* of the prescriptions given: What the language says should be mathematically and pedagogically definite, leaving no doubt of what the inner and outer boundaries are, of what is being asked of the student or teacher.

3. *Testability* of the lessons as described: The statement or demand, even if understandable and completely defined, might yet ask for results impossible to test in the school environment. We assign a positive value to testability.

For comparisons of clarity grades between the three Fordham Foundation math standards evaluations, see the Appendix beginning on page 123.

Content

Fig. 9: 2005 Grades for Content



[7] Much of this section is adapted from the "Criteria for Evaluation" section of *State Math Standards*, by Ralph A. Raimi and Lawrence Braden, Thomas B. Fordham Foundation, March 1998.

State average: 1.57
Range: 0.33-3.94

States to watch:
California (3.94)
Indiana (3.83)
Massachusetts (3.67)
Alabama (3.17)

States to shun:
Connecticut, Hawaii, Missouri (0.33)

Content, the second criterion, is plain enough in intent. Mainly, it is a matter of what might be called "subject coverage," i.e., whether the topics offered and the performance demanded at each level are sufficient and suitable. To the degree we can determine it from the standards documents, we ask, is the state asking K-12 students to learn the correct skills, in the best order and at the proper speed? For this report, the content score comprises 40 percent of the total grade for any state.

Here we separate the curriculum into three parts (albeit with fuzzy edges): Primary, Middle, and Secondary. It is common for states to offer more than one 9-12 curriculum, but also to print standards describing only the "common" curriculum, often the one intended for a universal graduation exam, usually in grade 11.

We cannot judge the division of content with year-by-year precision because few states do so, and we wish our scores to be comparable across states. As for the fuzziness of the edges of the three grade-span divisions, not even all those states with "elementary," "intermediate," and "high school" categories divide in the same way. One popular scheme is K-6, 7-9, and 10-12, while others divide it K-5, 6-8, and 9-12. In cases where states divide their standards into many levels (sometimes year-by-year), we shall use the first of these schemes. In other cases we accept the state's divisions and grade accordingly. Therefore, Primary, Middle, and Secondary will not necessarily mean the same thing from one state to another. There is really no need for such precision in our grading, though of course in any given curriculum it does make a difference where topics are placed.

Content gives rise to three criteria:

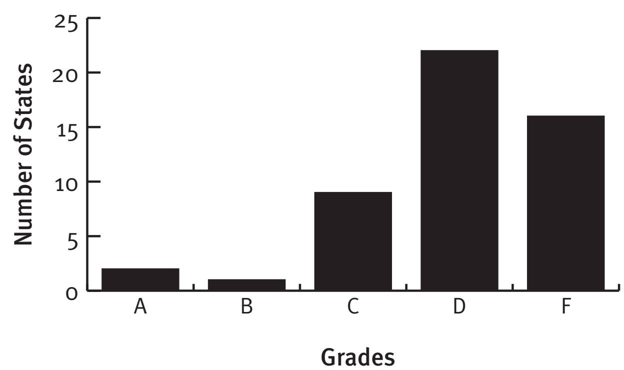
1. Primary school content (K-5, approximately)
2. Middle school content (or 6-8, approximately)
3. Secondary school content (or 9-12, approximately).

In many states, mathematics is mandatory through the tenth grade, while others might vary by a year or so. Our judgment of the published standards does not take account of what is or is not mandatory; thus, a rating will be given for secondary school content whether or not all students in fact are exposed to part or all of it. (Some standards documents only describe the curriculum through grade 11, and we adjust our expectations of content accordingly.)

For comparisons of content grades between the three Fordham Foundation math standards evaluations, see the Appendix beginning on page 123.

Reason

Fig. 10: 2005 Grades for Reason



State average: 1.15
Range: 0.00-4.00

States to watch:
Indiana (4.00)
California (3.83)
West Virginia (3.00)

States to shun:
Arkansas, Connecticut, Hawaii, Montana, New Hampshire, Oregon, Rhode Island, Wyoming (0.00)

Civilized people have always recognized mathematics as an integral part of their cultural heritage. Mathematics is the oldest and most universal part of our culture. In fact, we share it with all the world, and it has its roots in the most ancient of times and the most distant of lands.

The beauty and efficacy of mathematics derive from a common factor that distinguishes mathematics from the mere accretion of information, or application of practical skills and feats of memory. This distinguishing feature of mathematics may be called *mathematical reasoning*, reasoning that makes use of the structural organization by which the parts of mathematics are connected to each other, and not just to the real-world objects of our experience, as when we employ mathematics to calculate some practical result.

The essence of mathematics is its *coherent quality*. Knowledge of one part of a logical structure entails consequences that are inescapable and can be found out by reason alone. It is the ability to *deduce consequences* that would otherwise require tedious observation and disconnected experiences to discover, which makes mathematics so valuable in practice; only a confident command of the method by which such deductions are made can bring one the benefit of more than its most trivial results.

Should this coherence of mathematics be inculcated in the schools, or should it be confined to professional study in the universities? A 1997 report from a task force formed by the Mathematical Association of America to advise the National Council of Teachers of Mathematics in its revision of the 1989 NCTM Standards argues for its early teaching:

> *[T]he foundation of mathematics is reasoning. While science verifies through observation, mathematics verifies through logical reasoning. Thus the essence of mathematics lies in proofs, and the distinction among illustrations, conjectures and proofs should be emphasized. . . .*
>
> *If reasoning ability is not developed in the students, then mathematics simply becomes a matter of following a set of procedures and mimicking examples without thought as to why they make sense.*

Even a small child should understand how the memorization of tables of addition and multiplication for the small numbers (1 through 10) necessarily produces all other information on sums and products of numbers of any size whatever, once the structural features of the decimal system of notation are fathomed and applied. At a more advanced level, the knowledge of a handful of facts of Euclidean geometry—the famous Axioms and Postulates of Euclid, or an equivalent system—necessarily implies (for example) the useful Pythagorean Theorem, the trigonometric Law of Cosines, and a tower of truths beyond.

Any program of mathematics teaching that slights these interconnections doesn't just deprive the student of the beauty of the subject, or his appreciation of its philosophic import in the universal culture of humanity, but even at the practical level it burdens that child with the apparent need for memorizing large numbers of disconnected facts, where reason would have smoothed his path and lightened his burden. People untaught in mathematical reasoning are not being saved from something difficult; they are, rather, being deprived of something easy.

Therefore, in judging standards documents for school mathematics, we look to the "topics" as listed in the "content" criteria not only for their sufficiency, clarity, and relevance, but also for whether their statement includes or implies that they are to be taught with the explicit inclusion of information on their standing within the overall structures of mathematical reason.

A state's standards will not score higher on the Reason criterion just by containing a thread named "reasoning," "interconnections," or the like. It is, in fact, unfortunate that so many of the standards documents contain a thread called "Problem-solving and Mathematical Reasoning," since that category often slights the reasoning in favor of the "problem-solving," or implies that they are essentially the same thing. Mathematical reasoning is not found in the connection between mathematics and the "real world," but in the logical interconnections within mathematics itself.

Since children cannot be taught from the beginning "how to prove things" in general, they must begin with experience and facts until, with time, the interconnections of facts manifest themselves and become a subject of discussion, with a vocabulary appropriate to the level. Children must then learn how to prove certain particular things, memorable things, both as examples for reasoning and for the results obtained. The quadratic formula, the volume of a prism, and why the angles of a triangle add to a straight angle, are examples. What does the distributive law have to do with "long multiplication?" Why do independent events have probabilities that combine multiplicatively? Why is the product of two numbers equal to the product of their negatives?

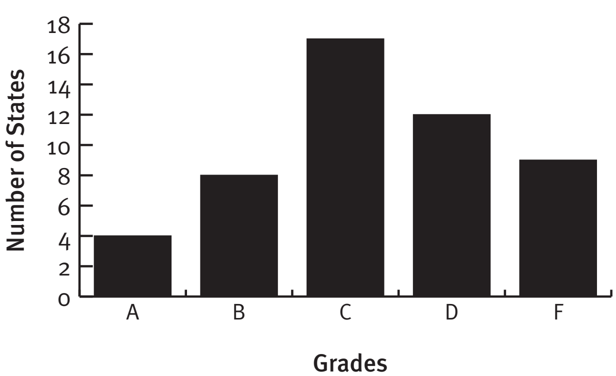
(At a more advanced level, the reasoning process can itself become an object of contemplation; but except for the vocabulary and ideas needed for daily mathematical use, the study of formal logic and set theory are not for K-12 classrooms.)

We therefore look at the *standards documents as a whole* to determine how well the subject matter is presented in an order, wording, or context that can only be satisfied by including due attention to this most essential feature of all mathematics.

For comparisons of reason grades between the three Fordham Foundation math standards evaluations, see the Appendix beginning on page 123.

Negative Qualities

Fig. 11: 2005 Grades for Negative Qualities



State average: 1.79
Range: 0.00-3.92

States to watch:
California (3.92)
Indiana (3.75)
Alabama, Massachusetts (3.50)
New Mexico, North Dakota (3.00)

States to shun:
Delaware, Washington (0.00)
Kansas (0.25)
Florida, Hawaii, Missouri (0.50)

This fourth criterion looks for the presence of unfortunate features of the document that contradict its intent or would cause its reader to deviate from what otherwise good, clear advice the document contains. We call one form of it *False Doctrine*. The second form is called *Inflation* because it offends the reader with useless verbiage, conveying no useful information. Scores for Negative Qualities are assigned a positive value; that is, a high score indicates the lack of such qualities.

Under False Doctrine, which can be either curricular or pedagogical, is whatever text contained in the standards we judge to be injurious to the correct transmission of mathematical information. To be sure, such judgments can only be our own, as there are disagreements among experts on some of these matters. Indeed, our choice of the term "false doctrine" for this category of our study is a half-humorous reference to its theological origins, where it is a synonym for heresy. Mathematics education has no official heresies, of course; yet if one must make a judgment about whether a teaching ("doctrine") is to be honored or marked down, deciding whether an expressed doctrine is true or false is necessary.

The NCTM, for example, prescribes the early use of calculators with an enthusiasm the authors of this report deplore, and the NCTM discourages the memorization of certain elementary processes, such as "long division" of decimally expressed real numbers, and the paper-and-pencil arithmetic of all fractions, that we think essential. We assure the reader, however, that our view is not merely idiosyncratic, but also has standing in the world of mathematics education.

While in general we expect standards to leave pedagogical decisions to teachers (as most standards documents do), so that pedagogy is not ordinarily something we rate in this study, some standards contain pedagogical advice that we believe undermines what the document otherwise recommends. Advice against memorization of certain algorithms, or a pedagogical standard mandating the use of calculators to a degree we consider mistaken, might appear under a pedagogical rubric. Then our practice of not judging pedagogical advice fails, for if the pedagogical part of the document gives advice making it impossible for the curricular part–as expressed there–to be accomplished properly, we must take note of the contradiction under this rubric of False Doctrine.

Two other false doctrines are excessive emphases on "real-world problems" as the main legitimating motive of mathematics instruction, and the equally fashionable notion that a mathematical question may have a multitude of different valid answers. Excessive emphasis on the "real-world" leads to tedious exercises in measuring playgrounds and taking census data, under headings like "Geometry" and "Statistics," in place of teaching mathematics. The idea that a mathematical question may have various answers derives from confusing a practical problem (whether to spend tax dollars on a recycling plant or a highway) with a mathematical question whose solution might form part of such an investigation. As the Mathematics Association of America Task Force on the NCTM Standards has noted,

> *[R]esults in mathematics follow from hypotheses, which may be implicit or explicit. Although there may be many routes to a solution, based on the hypotheses, there is but one correct answer in mathematics. It may have many components, or it may be nonexistent if the assumptions are inconsistent, but the answer does not change unless the hypotheses change.*

Constructivism, a pedagogical stance common today, has led many states to advise exercises in having children "discover" mathematical facts, algorithms, or "strategies." Such a mode of teaching has its value, in causing students to better internalize what they have learned; but wholesale application of this point of view can lead to such absurdities as classroom exercises in "discovering" what are really conventions and definitions, things that *cannot* be discovered by reason and discussion, but are arbitrary and must simply be learned.

Students are also sometimes urged to discover truths that took humanity many centuries to elucidate, such as the Pythagorean Theorem. Such "discoveries" are impossible in school, of course. Teachers so instructed will waste time, and end by conveying a mistaken impression of the standing of the information they must surreptitiously feed their students if the lesson is to come to closure. And often it all remains open-ended, confusing the lesson itself. Any doctrine tending to say that telling things to students robs them of the delight of discovery must be carefully hedged about with pedagogical information if it is not to be false doctrine, and unfortunately such doctrine is so easily and so often given injudiciously and taken injuriously that we deplore even its mention.

Finally, under False Doctrine must be listed the occurrence of plain mathematical error. Sad to say, several of the standards documents contain mathematical misstatements that are not mere misprints or the consequence of momentary inattention, but betray genuine ignorance.

Under the other negative rubric, *Inflation*, we speak more of prose than content. Evidence of mathematical ignorance on the part of the authors is a negative feature, whether or not the document shows the effect of this ignorance in its actual prescriptions, or contains outright mathematical error. Repetitiousness, bureaucratic jargon, or other evils of prose style that might cause potential readers to stop reading or paying attention, can render the document less effective than it should be, even if its clarity is not literally affected. Irrelevancies, such as the smuggling in of trendy political or social doctrines, can injure the value of a standards document by distracting the reader, even if they do not otherwise change what the standard essentially prescribes.

The most common symptom of irrelevancy, or evidence of ignorance or inattention, is bloated prose, the making of pretentious yet empty pronouncements. Bad writing in this sense is a notable defect in the collection of standards we have studied.

We thus distinguish two essentially different failures subsumed by this description of pitfalls, two Negative Qualities that might injure a standards document in ways not classifiable under the headings of Clarity and Content: Inflation (in the writing), which is impossible to make use of; and False Doctrine, which can be used but shouldn't.

For comparisons of Negative Qualities grades between the three Fordham Foundation math standards evaluations, see the Appendix beginning on page 123.